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LETTERS TO THE EDITORS

COMMENTS ON 'A HEAT TRANSFER PREDICTION METHOD FOR TURBULENT BOUNDARY LAYERS DEVELOPING OVER ROUGH SURFACES WITH TRANSPIRATION'

IN A RECENT paper, Ligrani *et al.* [1] presented a "closure method for the boundary layer equations which can be used to predict Stanton numbers, skin friction coefficients and mean profiles in boundary layers developing over rough surfaces". The purpose of this letter is to show that the proposed inner region mixing length expression for fully rough impermeable walls is essentially equivalent to that which would be deduced from Nikuradse's fully rough velocity profile, and thereby clarify the discussion presented in ref. [1].

For a fully rough impermeable flat plate the mixing length expression proposed in ref. [1] reduces to

$$l^+ = \kappa(y^+ + (\delta y_0)^+), \quad y^+ \geqslant 0, \tag{1}$$

with

$$(\delta y_0)^+ = 0.0307(Re_k - 46).$$

Reference [1] does not show explicitly how equation (1) was developed, or how it was used to calculate velocity profiles and skin friction: instead reference was made to refs. [2, 3]. However, these readers did not find ref. [2] helpful. In the description of the computer code, STAN 5, used to solve the complete boundary layer equations, the total shear stress is given as

$$\frac{g_c\tau}{\rho} = (\nu + \varepsilon_M)\frac{\partial U}{\partial y}, \quad \text{(ref. [2], equation (4–14))}, \tag{2}$$

and is the starting point for the development of equation (1). It is then assumed that the Couette flow approximation is valid in the inner region, and attention is restricted to an impermeable flat plate; so equation (2) thus becomes

$$u_\tau^2 = (\nu + \varepsilon_M)\frac{\partial U}{\partial y}, \quad \text{(ref. [2], equation (4–22))}. \tag{3}$$

Reference [2] then states "Next, the viscosity contribution in equation (4–22) is neglected, an approach justified by fully rough hydrodynamic data which show the same behavior regardless of molecular properties", and introduces the mixing length to obtain

$$u_\tau = l\frac{\partial U}{\partial y}, \quad \text{(ref. [2], equation (4–23))}. \tag{4}$$

The development of equation (1) proceeds from equation (4); however, we were unable to follow the reasoning used. We then decided to test equation (1) against experiment. A velocity profile was calculated by substituting equation (1) in equation (4) and integrating, but no agreement was found with the experimental data used in ref. [1]. We therefore decided to write a Letter to the Editors to express our reservations about the validity of the proposed mixing length model.

In their initial reply the authors of ref. [1] claimed that our use of equation (1) in equation (4) was incorrect, and rather equation (1) should be incorporated into a finite-difference boundary layer code in order to obtain correct results. We found this explanation of little value since it is well known that the Couette flow approximation is accurate for the inner region of a turbulent boundary layer, and thus simply using the full boundary layer equations cannot give a different result. Following further communications with the authors, in which we sought to discover exactly how equation (1) was incorporated into the STAN 5 computer code, the authors were able to identify the source of our dilemma. We were informed that in STAN 5 the kinematic viscosity is retained in equation (2), *even for fully rough walls*, and thus we should have used equation (3) rather than equation (4) for our Couette flow calculation. Equation (3) in nondimensional form is

$$1 = \left[1 + l^{+2}\left|\frac{du^+}{dy^+}\right|\right]\frac{du^+}{dy^+}. \tag{5}$$

Use of equation (1) for l^+ with $(\delta y_0^+) = 0.0307(Re_k - 46)$ requires that equation (5) be numerically integrated to obtain the velocity profile, which we did for $Re_k = 68.4$ (to correspond with the data of Pimenta *et al.* [4] used in ref. [1] for the purpose of comparing theory with experimental mean profiles). Table 1 shows that the agreement is indeed good.

But the question now arises as to why the viscosity should be retained in the boundary layer equations for a fully rough wall, particularly in view of the authors' statement quoted after equation (3) above, and in view of its omission from equation (4). Since viscosity should have no significant effect on a fully rough boundary layer, it is obvious that a key feature of the proposed mixing length expression, equation (1), is that it cancels the effect of viscosity close to the wall. To explore this

Table 1. Inner region dimensionless velocity profiles: $U_\infty = 27.1$ m s^{-1}, $x = 1.27$ m, $k_s = 0.79$ mm, $Re_k = 68.4$

		u^+		
y (mm)	$\ln(y/k_s)$	Experiment [4]	Equation (5)	Equation (8)
0.30	—	6.51	6.26	6.18
0.33	—	6.70	6.44	6.38
0.38	—	7.03	6.77	6.73
0.48	—	7.47	7.19	7.17
0.53	—	7.83	7.55	7.55
0.64	—	8.25	7.96	7.98
0.76	—	8.69	8.39	8.42
0.91	0.150	9.12	8.82	8.87
1.12	0.350	9.61	9.30	9.36
1.32	0.517	10.01	9.70	9.77
1.57	0.693	10.42	10.13	10.20
1.88	0.870	10.86	10.55	10.64
2.24	1.043	11.28	10.97	11.07
2.74	1.248	11.78	11.47	11.58
3.45	1.479	12.34	12.02	12.17
4.22	1.678	12.88	12.51	12.69
5.11	1.869	13.44	12.97	13.20
6.00	2.030	13.88	13.36	13.64

point further we rewrite equation (4) in dimensionless form as

$$du^+ = \frac{dy^+}{l^+}, \quad (6)$$

and with $l^+ = \kappa(y^+ + (\delta y_0)^+)$ integrate with $u^+ = 0$ at $y^+ = 0$ to give

$$u^+ = \frac{1}{\kappa} \ln \left(\frac{y^+}{(\delta y_0)^+} + 1 \right). \quad (7)$$

Use of $(\delta y_0)^+ = 0.0307 Re_k$, and $\kappa = 0.41$ gives

$$u^+ = \frac{1}{\kappa} \ln \left(\frac{y}{k_s} \right) + 8.5, \quad (8)$$

and the constant 8.5 is seen to be identical to Nikuradse's value (though Nikuradse did recommend $\kappa = 0.40$). Equation (8) is also shown in Table 1, and agrees with the velocity profile obtained from equation (5) with $(\delta y_0)^+ = 0.0307(Re_k - 46)$ to within 2%. Thus we conclude that if equation (3) is used to calculate the velocity profile, $(\delta y_0)^+ = 0.0307(Re_k - 46)$ is appropriate, while if equation (4) is used $(\delta y_0)^+ = 0.0307 Re_k$ is appropriate; also equation (1) does not contain any significant information beyond that which is contained in Nikuradse's fully rough velocity profile. The effect of the constant -46 in equation (1) is to approximately cancel the effect of ν in equations (2) and (3) so as to have a negligible effect of viscosity on the fully rough velocity profile. Thus we cannot agree with the assertion following equation (3) of ref. [1] that the implicit viscosity dependence in equation (1) results from viscosity not having a completely negligible effect on fully rough hydrodynamic behavior: the alleged viscosity dependence has been shown to be present simply because it is required to cancel the viscosity term in the boundary layer equations, *if that viscosity term is retained.* If the viscosity is not retained in the boundary layer equations for a fully rough wall, the constant -46 in equation (1) is not required, and the required mixing length offset can be deduced directly from Nikuradse's velocity profile, or its equivalent.

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REPLY TO "COMMENTS ON 'A HEAT TRANSFER PREDICTION METHOD FOR TURBULENT BOUNDARY LAYERS DEVELOPING OVER ROUGH SURFACES WITH TRANSPIRATION'"

MOST of the correct remarks made by Mills and Hang regarding the closure model of ref. [1] have been taken from our private communications with them. A reply is required in order to clarify their incorrect statements, and to avoid any confusion about the mixing-length closure scheme given in ref. [1]. It should also be mentioned that the use of equations (6)–(8) in the comment is a rearranged form of derivations presented in refs. [2, 3].

Referring to the last sentence of the comment, it is not logical to discard viscosity (viscous stress) terms if they are actually *non-negligible*. What is needed, and what is provided in ref. [1], is a viscosity-dependent mixing length which will reproduce the observed Reynolds number independence of the equation

$$U^+ = \frac{1}{\kappa} \ln (y/k_s) + 8.5. \quad (1)$$

As shown by results in refs. [2, 4], equation (1) is valid in log-regions of boundary layers developing over impermeable uniform spheres roughness when $Re_k > 55$. Reference [2] also shows that equation (1) is faithfully reproduced by the closure model described in ref. [1].

The viscosity independence of equation (1) results from near cancellation of the direct effects of viscosity on turbulent diffusion for an impermeable surface. However, Mills and Hang have confused the fact that this near cancellation is required because viscosity is present and non-negligible, even for Re_k values well above the fully rough threshold value of 55. The fact that viscosity is not insignificant is evident from results in Fig. 1, where ν/ε_M is shown as determined from measurements presented in refs. [2, 4]. These results are in excellent agreement with ν/ε_M from the mixing-length equations of ref. [1].

Towards the end of their comment, Mills and Hang have disagreed over a statement which we never made. In doing so, these individuals have missed two important points made clear after equation (3) in ref. [1]. The first is that the fully rough mixing length retains a viscosity dependence, which arises from the $Re_k' = 46$ term in equation (2) of ref. [1]. Secondly, the viscosity dependence of fully rough hydrodynamic properties, such as the mixing length and eddy diffusivity for momentum, ε_M, is expected to be most evident for roughness Reynolds numbers ranging from $Re_{k,R}$ to about 200.

We agree that, on *impermeable* rough walls, our viscosity-dependent mixing length almost exactly cancels the effect of the viscous shear stress, thus reproducing the observed Reynolds number independence of equation (1) for roughness Reynolds numbers above about 55. Clearly, the same result is obtained by ignoring both the mixing-length viscosity-dependent terms and the effect of viscosity on the diffusion of momentum. However, the cancellation is unlikely to persist in the case of transpiration for which our model was intended,